

BUOYANCY AND FLOTATION

What is buoyancy?

Let us consider a body immersed either fully or partially in a fluid. This body is subjected to an upward thrust (force) which tends to lift (or buoy) the body up. This tendency of the immersed body to be lifted up in the fluid due to upward force acting in a direction opposite to the direction of gravity is known as *buoyancy*.

What is buoyancy force?

The upward force tending to lift the body immersed in fluid is known as *buoyancy force* or *up thrust*.

What is centre of buoyancy?

The point of application of the buoyancy force on the body immersed in fluid is known as centre of buoyancy.

How can we evaluate the magnitude of buoyancy force?

By the Archimedes' principle. *What the Archimedes' principle states?* It states that when a body is immersed in a fluid either fully or partially, it is lifted up or buoyed up by a force whose magnitude is equal to the weight of the fluid displaced by the immersed body. For a body immersed in a fluid either fully or partially, there occurs an apparent loss in the weight of the body. *Why?* This is due to the upward force (buoyancy force or upthrust) acting on the body immersed in fluid.

Why the buoyancy force is exerted on the immersed body?

Due to various portions of the boundary surface of the immersed body being exposed to various pressure intensities of fluid in contact with the immersed body. This can be demonstrated as below.

Let us consider a body *ABCD* fully submerged in a liquid of specific weight, γ , as shown in Figure below. *What is the net horizontal pressure on the immersed body?* It is equal to zero. *Why?* Because, the horizontal pressure on the vertical projection from both sides are equal in magnitude and opposite in direction. *What is the resultant force acting on the immersed body in the vertical direction?* Let us consider an elementary vertical prism *MN* having a cross-sectional area *dA* out of the whole body. Let the pressure intensity acting on the top end, *M*, of the

elementary prism MN be p_1 . The pressure force due to this pressure intensity on the top end of the prism is $(p_1 dA)$. This pressure force acts in the vertical downward direction. Let the pressure intensity on the bottom end, N , of the prism, MN , be p_2 . The pressure force due to this pressure intensity on the bottom end of the prism is $(p_2 dA)$. This pressure force acts in the vertical upward direction. Pressure intensity, $p_1 = \gamma h_1$ and pressure intensity, $p_2 = \gamma h_2$. Let $(h_2 - h_1) = y$. γ is the specific weight of the fluid. We know that $p_2 > p_1$. Hence, the difference between the upward force, $p_2 dA$ and the downward force, $p_1 dA$ is a net upward force as $p_2 > p_1$. This net upward force is equal to the buoyant force dF_B on the vertical prism MN .

$$dF_B = (p_2 dA - p_1 dA) = [(\gamma h_2) dA - (\gamma h_1) dA] = \gamma (h_2 - h_1) dA = \gamma y dA \quad \dots\dots (1)$$

Let \bar{dV} represents the volume of the vertical prism \overline{MN} , then $\bar{dV} = y dA$, where y is the height of the prism MN and dA is the cross-sectional area of the prism MN . Substituting $y dA = \bar{dV}$ in equation (1), we have,

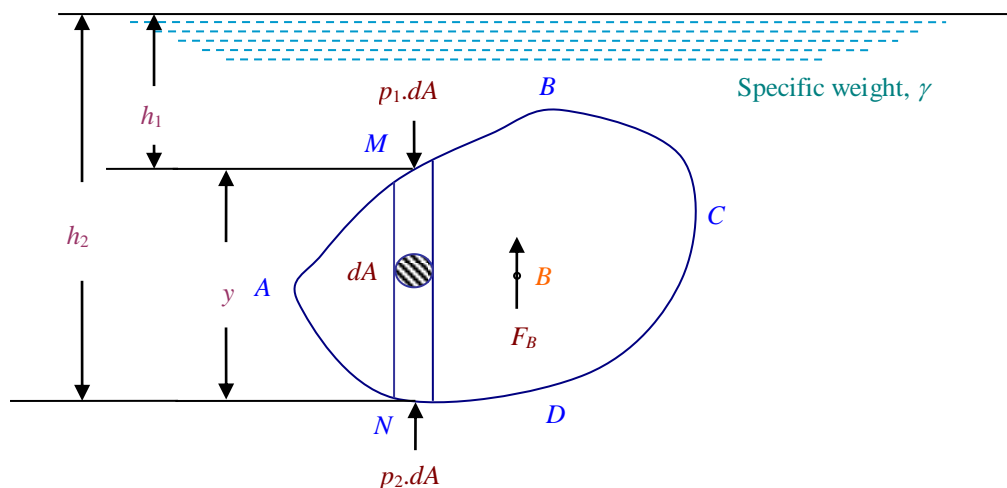
$$dF_B = \gamma \bar{dV} \quad \dots\dots (2)$$

The buoyancy force F_B on the entire submerged body $ABCD$ is obtained by integrating (2)

$$F_B = \int dF_B = \int \gamma \bar{dV} = \gamma \bar{V} \quad \dots\dots (3)$$

where \bar{V} = volume of the submerged body. Also, \bar{V} indicates the volume of water displaced by the submerged body. It should be noted that the volume of water displaced is equal to the volume of the submerged portion of the body. In this case as the entire body is submerged in water, volume of water displaced is equal to the volume of body. Now, weight of water displaced by the submerged body is equal to the product of the specific weight of water and the volume of water displaced. Hence, from equation (3), it can be stated that the buoyancy force exerted on the submerged body is equal to the weight of fluid displaced by the body.

The buoyancy force acts vertically in the upward direction through the centre of buoyancy. The centre of buoyancy evidently coincides with the centre of gravity of the volume of water (fluid) displaced. It should be noted that for a fully submerged body of homogeneous composition, the centre of buoyancy will coincide with the centre of gravity of the body.



Figure

Let us consider the case of a body floating at a surface of separation (or interface) between two immiscible fluids of specific weights γ_1 and γ_2 as shown in Figure below. Let us evaluate the magnitude of the buoyancy force dF acting on an elementary vertical prism MN of cross-sectional area dA . As expressed in equation (1), we have

$$dF_B = (p_2 dA - p_1 dA)$$

p_2 = pressure intensity at bottom end N of vertical prism MN
 = (pressure intensity due to column $(h_1 + y_1)$ of upper liquid of specific weight γ_1) + (pressure intensity due to column y_2 of lower liquid of specific weight γ_2)

$$= \gamma_1 (h_1 + y_1) + \gamma_2 y_2$$

p_1 = pressure intensity at top end M of vertical prism MN
 = pressure intensity due to column h_1 of liquid of specific weight γ_1

$$\begin{aligned} \text{Hence, } dF_B &= [\{\gamma_1 (h_1 + y_1) + \gamma_2 y_2\} dA - \{\gamma_1 h_1\} dA] \\ &= \gamma_1 h_1 dA + \gamma_1 y_1 dA + \gamma_2 y_2 dA - \gamma_1 h_1 dA \\ &= (\gamma_1 y_1 + \gamma_2 y_2) dA \end{aligned} \quad \dots\dots (4)$$

$y_1 dA$ = volume of portion of elementary vertical prism MN , $d\bar{V}_1$ submerged in upper liquid of specific weight γ_1 above the interface of the two immiscible liquids.

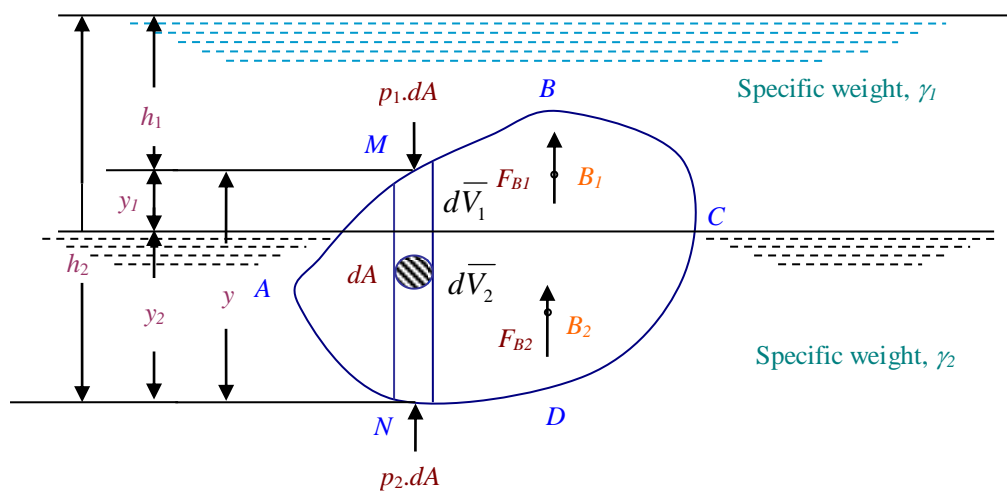
$y_2 dA$ = volume of portion of elementary vertical prism MN , $d\bar{V}_2$ submerged in lower liquid of specific weight γ_2 below the interface of the two immiscible liquids.

$$\text{Hence, } dF_B = \gamma_1 d\bar{V}_1 + \gamma_2 d\bar{V}_2 \quad \dots\dots (5)$$

The buoyancy force F_B on the entire submerged body $ABCD$ is obtained by integrating (5)

$$F_B = \int dF_B = \int \gamma_1 d\bar{V}_1 + \gamma_2 d\bar{V}_2 = \gamma_1 \bar{V}_1 + \gamma_2 \bar{V}_2 \quad \dots\dots (6)$$

where \bar{V}_1 is the volume of portion of body submerged in upper liquid of specific weight γ_1 and \bar{V}_2 is the volume of portion of body submerged in lower liquid of specific weight γ_2 . Also,



Figure

Example. A rectangular pontoon has a width B of 6 m, a length l of 12 m, and a draught D of 1.5 m in fresh water of density 1000 kg m^{-3} . Calculate (a) the weight of the pontoon, (b) its draught in sea water of density 1025 kg m^{-3} and (c) the load in kN that can be supported by the pontoon in fresh water if the maximum draught permissible is 2 m.

Solution.

What is a pontoon?

A pontoon is a flat-bottomed boat.

What is draught?

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Topic: **Buoyancy and Flotation**

Draught refers to the depth of portion of pontoon immersed in water.

Data given: Width of pontoon, $B = 6 \text{ m}$

Length of pontoon, $l = 12 \text{ m}$

Draught of pontoon floating in fresh water, $D = 1.5 \text{ m}$

Mass density of fresh water, $\rho_{\text{water}} = 1000 \text{ kg m}^{-3}$

Mass density of sea water, $\rho_{\text{sea water}} = 1025 \text{ kg m}^{-3}$

Maximum permissible draught in fresh water, $D_{\text{max}} = 2 \text{ m}$

Required: Weight of pontoon, W

Draught of pontoon when floating in sea water, D_1

Load supported by pontoon floating in fresh water, W_1

Let us consider the case of pontoon floating in fresh water without any load supported on it.

Upthrust on immersed portion of pontoon = Weight of pontoon

Further, we have,

Upthrust on immersed portion of pontoon = Weight of liquid displaced by
the pontoon

Hence,

Weight of pontoon = Weight of liquid displaced by the pontoon

Weight of liquid displaced by the pontoon =

(Volume of liquid displaced by the pontoon) x
(Weight density of liquid)

= (Volume of portion of pontoon immersed in liquid) x
(Weight density of liquid)

Volume of portion of pontoon immersed in liquid = (Width of pontoon) x
(Length of pontoon) x (Draught of pontoon in fresh water)
= BID

Weight density of liquid = (Mass density of liquid) x
(acceleration due to gravity)

Hence, Weight of liquid displaced by the pontoon = $(BID) \times (\rho g)$

\Rightarrow Weight of pontoon, $W = 6 \text{ m} \times 12 \text{ m} \times 1.5 \text{ m} \times 1000 \text{ kg m}^{-3} \times 9.81 \text{ ms}^{-2}$
 $= 1059480 \text{ N} = 1059.48 \text{ kN}$

When the pontoon of weight $W = 1059.5 \text{ N}$ is floating in sea water of mass density 1025 kg m^{-3} , we have,

$W = (BID_1) \times (\rho_{\text{sea water}} \cdot g)$

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$$\Rightarrow 1059.48 \text{ kN} = (6 \text{ m} \times 12 \text{ m} \times D_1) \times (1025 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2})$$

$$\Rightarrow D_1 = \frac{1059.48 \times 10^3 \text{ N}}{(6 \text{ m} \times 12 \text{ m} \times 1025 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2})} = 1.463 \text{ m}$$

When the draught is maximum at 2 m in fresh water,

$$\begin{aligned} \text{Total upthrust on pontoon} &= \text{Weight of water displaced} \\ &= (\text{Weight density of fresh water}) \times (\text{Volume of water displaced by pontoon}) \\ &= (\text{Weight density of fresh water}) \times (\text{Volume of immersed portion of pontoon}) \\ &= (\rho g) \times (BID_{max}) \\ &= 1000 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2} \times 6 \text{ m} \times 12 \text{ m} \times 2 \text{ m} \\ &= 1412640 \text{ N} = 1412.64 \text{ kN} \end{aligned}$$

Load that can be supported by the pontoon =

$$\begin{aligned} &\text{Total upthrust on the pontoon} - \text{Weight of pontoon} \\ &= 1412.64 \text{ kN} - 1059.48 \text{ kN} \\ &= 353.16 \text{ kN} \end{aligned}$$

Example. A steel pipeline conveying gas has an internal diameter of 1.2 m and an external diameter 1.25 m. It is laid across the bed of a river, completely immersed in water and is anchored at intervals of 3 m along its length. Calculate the buoyancy force in newtons per metre and the upward force in newtons on each anchorage. Density of steel = 7900 kg m⁻³, density of water = 1000 kg m⁻³.

Solution.

Data given: Internal diameter of steel pipeline, $D_{int} = 1.2 \text{ m}$
External diameter of steel pipeline, $D_{ext} = 1.25 \text{ m}$
Length of pipeline between successive anchorages, $l = 3 \text{ m}$
Density of steel, $\rho_{steel} = 7900 \text{ kg m}^{-3}$
Density of water, $\rho = 1000 \text{ kg m}^{-3}$

Required: Buoyancy force in newtons per metre
Upward force in newtons on each anchorage

Weight of steel pipeline between any two successive anchorages,
 $W = (\text{Volume of steel pipeline between any two successive anchorages}) \times (\text{Weight density of steel})$

Volume of steel pipeline between any two successive anchorages =
 $(\text{Area of cross-section of steel pipeline}) \times (\text{Length of pipeline between two successive anchorages})$

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$$= \frac{\pi}{4} (D_{ext}^2 - D_{int}^2) \times l$$

$$= \frac{\pi}{4} (1.25^2 - 1.2^2) \times 3$$

$$= 0.28875 \text{ m}^3$$

Weight density of steel = (Mass density of steel) x
(acceleration due to gravity)

$$= \rho_{steel} \cdot g$$

$$= 7900 \times 9.81$$

$$= 77499 \text{ N m}^{-3}$$

Hence, $W = 0.28875 \text{ m}^3 \times 77499 \text{ N m}^{-3}$

$= 22378 \text{ N}$ per 3 m length of pipeline between two successive anchorages

$= 7459.3 \text{ N m}^{-1}$ length of pipeline

Volume of water displaced by steel pipeline of length 3 m = (area of cross-section of flow passage of steel pipeline including the thickness of steel pipeline) x (length of pipeline between two successive anchorages)

$$= \left(\frac{\pi}{4} D_{ext}^2 \right) l$$

$$= \left(\frac{\pi}{4} \times 1.25^2 \right) \times 3$$

$$= 3.683 \text{ m}^3$$

Weight of water displaced by pipeline of length 3 m = (Volume of water displaced by steel pipeline of length 3 m) x (Weight density of water)

$$= (3.683 \text{ m}^3) \times (1000 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2})$$

$$= 36130 \text{ N}$$

Buoyant force on 3 m length of steel pipeline =

$$\text{Weight of water displaced by pipeline of 3 m length} \\ = 36130 \text{ N}$$

Buoyant force on 1 m length of steel pipeline = $36130 \text{ N} / 3 \text{ m}$

$$= 12043 \text{ N m}^{-1}$$

Upward thrust on each anchorage = (Buoyant force on 3 m length of steel pipeline between two successive anchorages) –
(Weight of 3 m length of steel pipeline between two successive anchorages)

$$= 36130 \text{ N} - 22378 \text{ N}$$

$$= 13752 \text{ N}$$

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Example. The ball-operated valve shown in figure below controls the flow from a tank through a pipe to a lower tank, in which it is situated. The water level in the upper tank is 7 m above the 10 mm diameter valve opening. Calculate the volume of the ball which must be submerged to keep the valve closed.

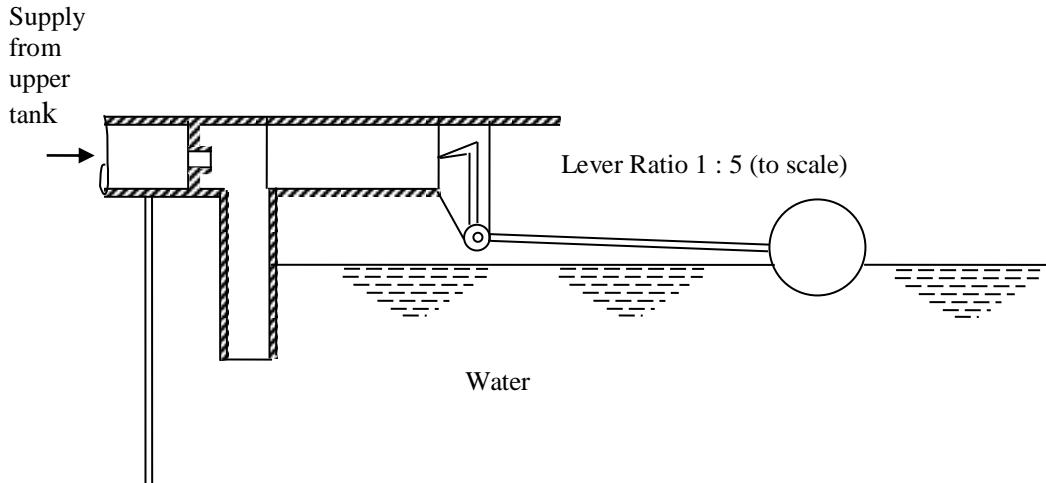


Figure 1

Solution.

Data given: Height of water surface in upper tank above the centreline of valve opening that leads water to the lower tank from the upper tank, $H = 7\text{ m}$

Diameter of valve opening, $d = 10\text{ mm} = 10 \times 10^{-3}\text{ m}$

Required: Volume of the ball which must be submerged to keep the valve closed, V

Pressure intensity at the centre line of valve opening due to the head of $H = 7\text{ m}$ of water above it in the upper tank is given by,

$$p = \gamma H$$

where $\gamma =$ specific weight or weight density of water $= 9810\text{ N m}^{-3}$

$$p = (9810\text{ N m}^{-3}) \times (7\text{ m}) \\ = 68870\text{ N m}^{-2}$$

Horizontal pressure force exerted on the valve opening along its centre line is given by,

$$P_H = (\text{pressure intensity due to } 7\text{ m head of water above the valve opening}) \times (\text{cross-sectional area of valve opening})$$

$$\begin{aligned} &= (68870 \text{ N m}^{-2}) \times \left(\frac{\pi}{4} d^2 \right) \\ &= 68870 \times \left(\frac{\pi}{4} \times 0.01^2 \right) \\ &= 5.411 \text{ N} \end{aligned}$$

It is required to determine the volume of portion of ball that must be submerged in water in the lower tank in order to keep the valve opening closed. To determine this, we have to evaluate the buoyant force required to act on the ball in order to keep the valve opening closed. This buoyant force exerted on the ball due to submergence of portion of the ball must be capable of resisting the horizontal pressure force, P_H , of magnitude 5.411 N exerted on the valve opening. The lever ratio, to scale, is given to be 1:5. That is, for resisting a force of 5.411 N, in order to keep the valve opening closed, the lever mechanism must exert a horizontal force of magnitude $\left[5.411 \text{ N} \times \left(\frac{1}{5} \right) \right] = 1.0822 \text{ N}$, in the opposite direction. This horizontal and opposite force of magnitude 1.0822 N must be equal to the magnitude of the buoyant force, F_B , exerted in the upward direction on the ball shown in Figure.

$$\begin{aligned} \text{Buoyant force, } F_B &= \text{Weight of water displaced due to submerged portion of ball} \\ \Rightarrow 1.0822 \text{ N} &= (\text{Specific weight of water}) \times (\text{volume of submerged portion of ball}) \\ &= \gamma \times (\text{volume of submerged portion of ball}) \\ \Rightarrow \text{Volume of submerged portion of ball} &= \frac{1.0822 \text{ N}}{9810 \text{ N m}^{-3}} = 0.00011 \text{ m}^3 \\ &= 0.00011 \times (100 \text{ cm})^3 \\ &= 110 \text{ cm}^3 \end{aligned}$$

Stability of Floating Bodies

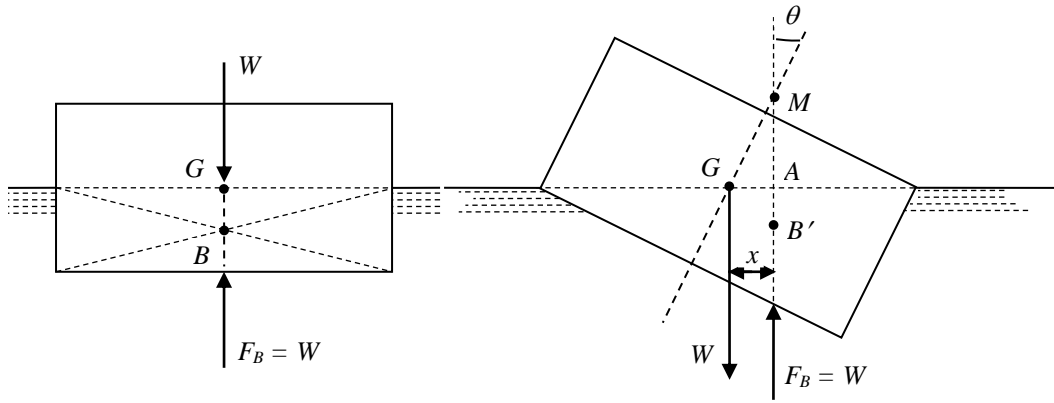


Figure (a)

Figure (b)

Stable equilibrium

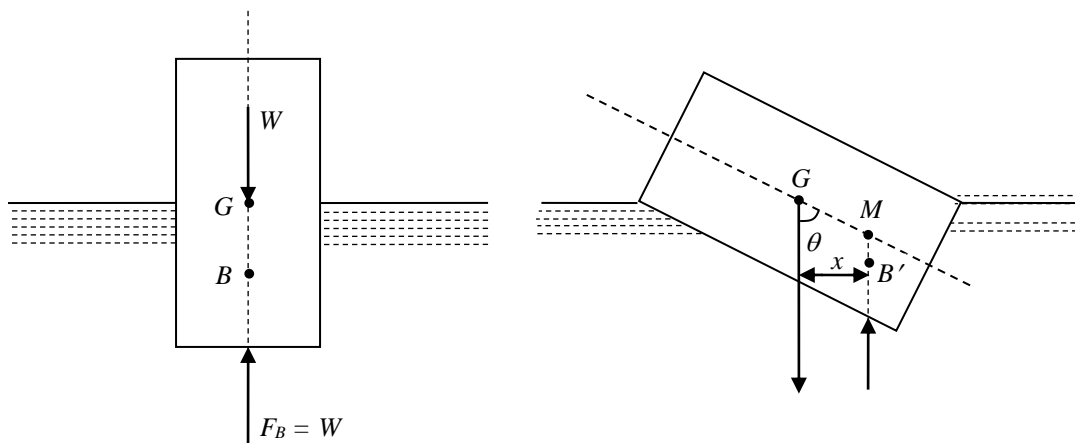


Figure (c)

Figure (d)

Unstable equilibrium

Figure (a) shows a body floating in equilibrium. The weight W of the body acts through the centre of gravity G of the body. The buoyant force (upthrust) F_B acts through the centre of buoyancy B . The centre of buoyancy is nothing but the centre of gravity of the displaced fluid. As the body is in vertical equilibrium, the line of action of F_B is in the same straight line as W . When the body is displaced through an angle θ , the weight W of the body continues to act through the centre of gravity G of the body; the volume of liquid displaced remains the same since F_B is equal to W , but the shape of this volume changes and the centre of gravity of the volume of fluid displaced in the tilted position, which is the centre of buoyancy in the tilted position, moves relative to the body from B (in vertical equilibrium of the body) to B' . In the tilted position of the body, as the lines of action of F_B and W are not in the same straight line, a turning moment proportional to $W \times \theta$ is produced. In Figure (b), the

turning moment caused is a righting moment while in Figure (d), it is an overturning moment. Let M be the point at which the line of action of the buoyant force F_B cuts the original vertical through the centre of gravity G of the body.

$$\begin{aligned}\text{From right – angled triangle } GAM, \sin \angle AMG &= \sin \theta \\ &= \text{opposite/hypotenuse} \\ &= AG/GM = x/GM\end{aligned}$$

$$\Rightarrow x = GM \sin \theta$$

If the angle of tilt θ is small, $\sin \theta = \tan \theta = \theta$ in radians.

The point M is called the metacentre and the distance GM is called the metacentric height.

In Figure (b), the metacentre M lies above the centre of gravity G , the turning moment equal to $W \times x = W \times GM \times \sin \theta = W \times GM \times \theta$ is produced. This turning moment is a righting moment as it brings the body back to the vertical upright position (vertical equilibrium). Here, the body is in *stable equilibrium*, and the metacentric height GM is regarded as positive.

In Figure (d), the metacentre M lies below the centre of gravity G of the body, the turning moment equal to $W \times x = W \times GM \times \sin \theta = W \times GM \times \theta$ is produced. This turning moment is an overturning moment as it causes the body to sink continuously and the body is said to be in *unstable equilibrium*. The metacentric height GM is regarded as negative.

If the metacentre M coincides with the centre of gravity G of the body, the body is said to be in *neutral equilibrium*.

Since a floating body can tilt in any direction, it is usual, for a ship, to consider displacement about the longitudinal (rolling) and transverse (pitching) axes. The position of the metacentre and the value of metacentric height will generally be different for rolling and pitching.

Determination of the Metacentric Height

The metacentric height of a floating vessel can be determined if the angle of tilt caused by moving a load P (Figure) through a known distance x across the deck is measured.

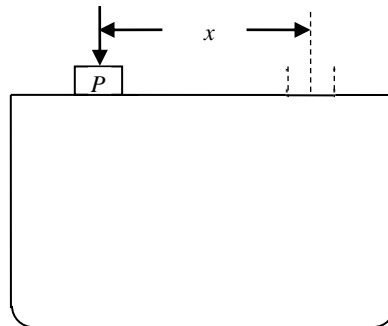


Figure Determination of metacentric height

Overtopping moment due to movement of load $P = P \times x$

If GM is the metacentric height and W is the total weight of the vessel including P , then,

Righting moment = $W \times GM \times \theta$

For equilibrium in the tilted position, the righting moment must equal the overturning moment so that,

$$W \times GM \times \theta = P \times x$$

Metacentric height, $GM = Px/W\theta$

The true metacentric height is the value of GM as $\theta \rightarrow 0$.

Determination of the Position of Metacentre Relative to the Centre of Buoyancy

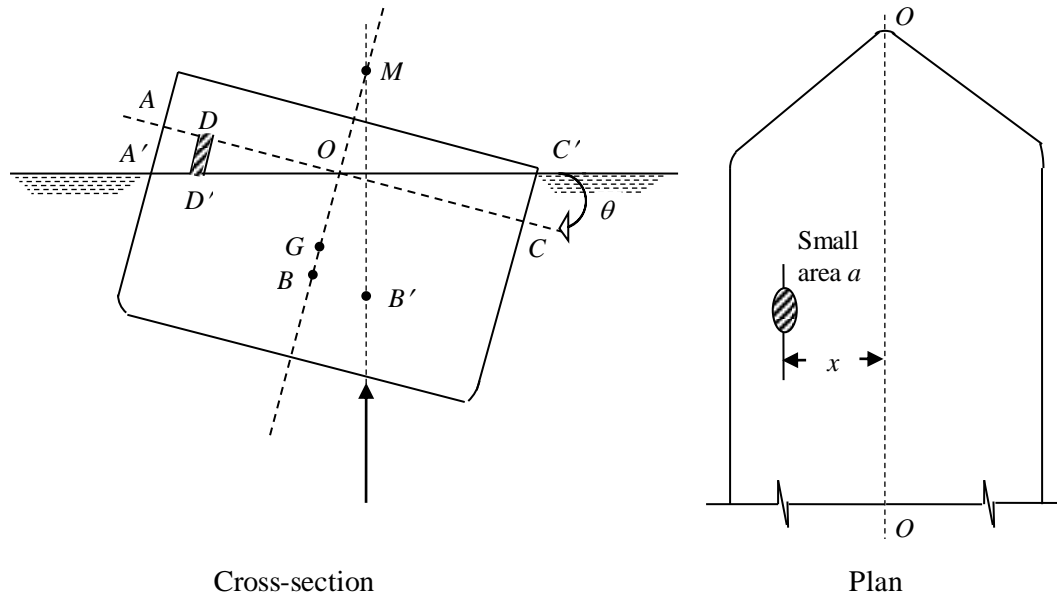


Figure Height of metacentre above centre of buoyancy

For a vessel of known shape and displacement, the position of the centre of buoyancy B and the position of metacentre M relative to the centre of buoyancy B can be calculated as follows.

In figure above, AC is the original waterline plane and B is the centre of buoyancy in the equilibrium position of the vessel. Let the vessel be tilted through a small angle θ . When the vessel is tilted through this small angle θ , the centre of buoyancy would move to B' due to the alteration in the shape of the displaced fluid. In the displaced position of the vessel, $A'C'$ is the waterline plane.

From figure, $\angle BMB' = \theta$, let the line joining the centre of buoyancy B in the equilibrium position and the centre of buoyancy B' in the tilted position be assumed to be perpendicular to the line joining the centre of buoyancy B in the equilibrium position and the metacentre M . Then, the line joining the centre of buoyancy B' in the tilted position of the vessel and the metacentre M becomes the hypotenuse in the right-angled triangle $B'BM$. Hence, from right-angled triangle $B'BM$,

$$\tan \angle BMB' = \tan \theta = \text{opposite side} / \text{adjacent side} = BB'/BM$$

When the tilt θ is small, $\tan \theta \approx \theta = BB'/BM$

$$\Rightarrow BM = BB'/\theta$$

The movement of the centre of buoyancy from B to B' is due to the removal of the volume of fluid corresponding to the wedge AOA' and the addition of the volume of fluid corresponding to the wedge COC' . It should be noted that centre of buoyancy refers to the centre of gravity of the displaced fluid. As the weight of the fluid displaced is equal to the weight of the vessel, even in the tilted position, the weight of the fluid displaced by the vessel remains unchanged. Hence,

$$\text{Weight of volume of fluid contained in wedge } AOA' = \text{Weight of volume of fluid contained in wedge } COC'$$

If a is a small area in the waterline plane at a distance x from the axis of rotation OO , it will generate a small volume, shown shaded, when the vessel is tilted.

$$\text{Volume swept out by the small area } a \text{ as the vessel is tilted} = DD' \times a$$

$$\text{From Figure } \angle DOD' = \theta = \angle COC'$$

θ is the subtended angle at the axis of rotation; DD' is the arc traced by the small area a when the vessel is tilted through the small angle θ , x is the radial distance of the small area a from the axis of rotation OO . Hence,

$$\theta = \text{arc/radial distance} = DD'/x \\ \Rightarrow DD' = x\theta$$

Therefore, volume swept out by the small area a as the vessel is tilted becomes equal to $a(x\theta)$.

Considering such small areas a in the waterline plane from the axis of rotation O to the left edge A of the vessel and summing up all volumes generated by such small areas a as the vessel is tilted, the volume of wedge AOA' is obtained. The weight of fluid represented by the volume of wedge AOA' is obtained by multiplying the volume of wedge AOA' by the specific weight γ of fluid.

$$\text{Weight of wedge } AOA' = \Sigma \gamma(ax\theta)$$

Here, x runs from 0 to OA .

Similarly, weight of wedge $COC' = \Sigma \gamma(ax\theta)$

Here, x runs from 0 to OC .

As weight of wedge AOA' is equal to weight of wedge COC' , we have,

$$\begin{aligned}\Sigma \gamma(ax\theta) &= \Sigma \gamma(ax\theta) \\ \Rightarrow \gamma\theta \Sigma ax &= \gamma\theta \Sigma ax\end{aligned}$$

Σax is the first moment of area of the waterline plane about the axis of rotation OO ; therefore, the axis of rotation OO must pass through the centroid of the waterline plane.

Calculation of distance BB'

The couple produced by the movement of wedge AOA' to COC' must be equal to the couple due to the movement of F_B from B to B' .

Moment about OO of the weight of fluid swept out by area $a = \gamma(ax\theta) \times x$
Total moment due to altered displacement = $\gamma\theta \Sigma ax^2$

Σax^2 is the second moment of area of the waterline plane about the axis of rotation OO , putting $\Sigma ax^2 = I$, the total moment due to altered displacement becomes $\gamma\theta I$.

Moment due to movement of $F_B = F_B \times BB' = \gamma V \times BB'$

where V = volume of fluid displaced

Equating () and (), we have,

$$\begin{aligned}\gamma V \times BB' &= \gamma\theta I \\ \Rightarrow BB' &= \gamma\theta I / \gamma V = \theta I / V \\ BM &= BB' / \theta = I / V\end{aligned}$$

The distance BM is known as the *metacentric radius*.

Example. A cylindrical buoy 1.8 m in diameter, 1.2 m high and weighing 10 kN floats in salt water of density 1025 kg m^{-3} . Its centre of gravity is 0.45 m from the bottom. If a load of 2 kN is placed on the top, find the

maximum height of the centre of gravity of this load above the bottom if the buoy is to remain in stable equilibrium.

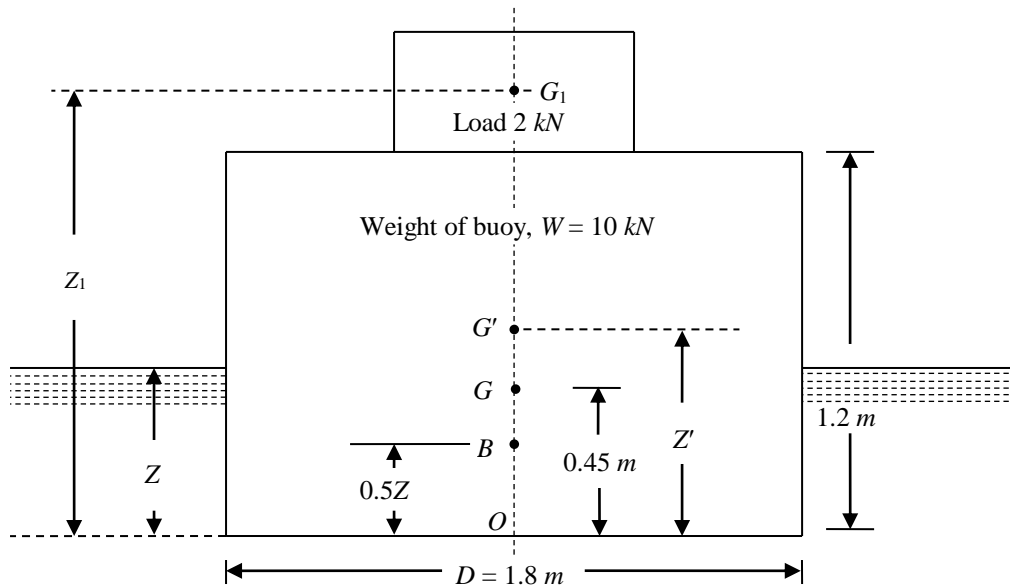


Figure 2

Solution.

In Figure 2, let G be the centre of gravity of the buoy. Let G_1 be the centre of gravity of the 2 kN load placed on the top of the buoy. G_1 is located at a vertical height Z_1 from the bottom of the buoy. Let G' be the combined centre of gravity of the buoy and the 2 kN load. G' is located at a vertical height Z' from the bottom of the buoy.

When the 2 kN load is in position, let V be the volume of salt water displaced and let Z be the depth of immersion of the buoy.

Buoyant force on the buoy, $F_B =$ Weight of salt water displaced by the buoy

$$\begin{aligned} \Rightarrow F_B &= (\text{specific weight of salt water}) \times (\text{volume of salt water displaced}) \\ &= [(\text{mass density of salt water, } \rho) \times (\text{acceleration due to gravity, } g)] \\ &\quad \times (\text{volume of salt water displaced, } V) \\ &= [(1025\text{ kg m}^{-3}) \times (9.81\text{ m s}^{-2})] \times V \end{aligned}$$

Volume of salt water displaced, V

$$\begin{aligned} &= \text{volume of portion of cylindrical buoy immersed in salt water} \\ &= (\text{Area of base of cylindrical buoy}) \times \\ &\quad (\text{depth of portion of cylindrical buoy immersed in salt water}) \\ &= \left(\frac{\pi}{4} D^2\right) Z \end{aligned}$$

$$\text{Hence, } F_B = \rho g \left(\frac{\pi}{4} D^2 \right) Z$$

For stable equilibrium of the system (cylindrical buoy with load of 2 kN on its top), the buoyant force must be equal to the combined weight of the cylindrical buoy and the 2 kN load at its top. Let W be the weight of the cylindrical buoy. Let the 2 kN load be denoted as W_1 . Hence, we have,

$$F_B = W + W_1 = \rho g \left(\frac{\pi}{4} D^2 \right) Z$$

From the above equation, we have, the depth of immersion of the buoy, Z , as

$$Z = \frac{W + W_1}{\rho g \left(\frac{\pi}{4} D^2 \right)} = \frac{4(W + W_1)}{\rho g \pi D^2} = \frac{4(10000 \text{ N} + 2000 \text{ N})}{1025 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2} \times \pi \times (1.8 \text{ m})^2} = 0.469 \text{ m}$$

The centre of buoyancy B will be located at the centre of gravity of the displaced water, so that, $\overline{OB} = \frac{Z}{2} = \frac{0.469}{2} = 0.234 \text{ m}$

If the buoy and the 2 kN load are just in stable equilibrium, the metacentre M just coincides with the combined centre of gravity G' of the buoy and the 2 kN load on its top. The metacentric height $G'M$ will then be zero (as M coincides with G) and $\overline{BG'} = \overline{BM}$.

$$\text{We have, } \overline{BG'} = \overline{BM} = \frac{I}{V} = \frac{\frac{\pi}{64} D^4}{\left(\frac{\pi}{4} D^2 \right) Z} = \frac{D^2}{16Z} = \frac{1.8^2}{16 \times 0.469} = 0.432 \text{ m}$$

Thus, the vertical height, Z' , of the combined centre of gravity, G' , of the cylindrical buoy and the 2 kN load, from the base of the buoy is given by

$$Z' = \overline{OB} + \overline{BG'} = 0.5Z + 0.432 = (0.5 \times 0.469) + 0.432 = 0.667 \text{ m}$$

The value of Z_1 corresponding to this value of Z' is found by taking moments the forces about O :

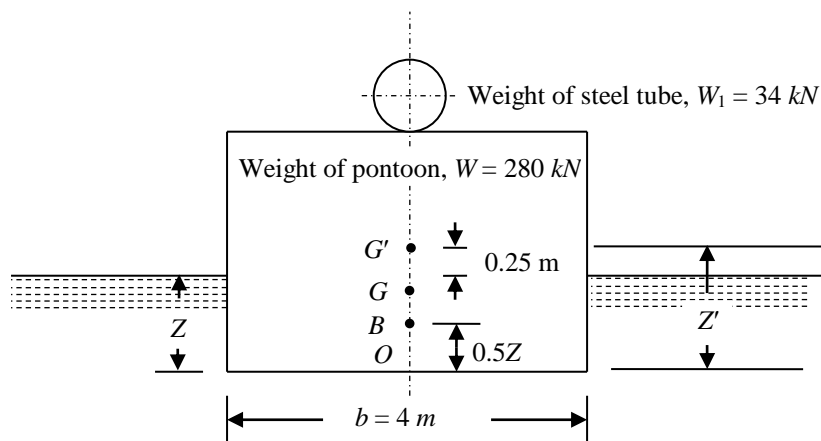
The weight of the cylindrical buoy, W , acts through the centre of gravity, G , of the cylindrical buoy. G is located at a vertical height of 0.45 m from O on the base of the buoy. The weight of the load, $W_1 = 2 \text{ kN}$, acts through the centre of gravity of the load, G_1 , located at a vertical distance Z_1 from O . The combined weight, $W + W_1$, of the cylindrical buoy and the 2 kN load, acts through the combined centre of gravity, G' , located at a vertical distance $Z' = 0.667 \text{ m}$ from O .

Now, taking moments of forces about O , we have,

$$\begin{aligned}
 W(0.45 \text{ m}) + W_1 Z_1 &= (W + W_1) Z' \\
 \Rightarrow (10000 \text{ N} \times 0.45 \text{ m}) + (2000 \text{ N} \times Z_1) &= (10000 \text{ N} + 2000 \text{ N}) \times (0.667 \text{ m}) \\
 \Rightarrow 4500 \text{ N m} + (2000 \text{ N} \times Z_1) &= 8004 \text{ N m} \\
 \Rightarrow Z_1 = \frac{8004 \text{ N m} - 4500 \text{ N m}}{2000 \text{ N}} &= 1.752 \text{ m}
 \end{aligned}$$

Example. A rectangular pontoon 10 m by 4 m in plan weighs 280 kN. A steel tube weighing 34 kN is placed longitudinally on the deck. When the tube is in central position, the centre of gravity for the combined weight lies on the vertical axis of symmetry 250 mm above the water surface. Find the (a) metacentric height (b) maximum distance the tube may be rolled laterally across the deck if the angle of heel is not to exceed 5° .

Solution.



In Figure 3, let G be the centre of gravity of the rectangular pontoon. Let G' be the combined centre of gravity of the rectangular pontoon and the cylindrical steel tube. G' is located at a vertical height Z' from the bottom of the buoy. Z is the depth of immersion of the rectangular pontoon. The vertical height of the combined centre of gravity of the rectangular pontoon and the steel tube is 0.25 m from the water surface. Let V be the volume of water displaced.

Buoyant force on the buoy, $F_B =$ Weight of water displaced by the rectangular pontoon

$$\Rightarrow F_B = (\text{specific weight of water, } \gamma) \times (\text{volume of water displaced, } V)$$

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$$\begin{aligned} &= [(\text{mass density of water, } \rho) \times (\text{acceleration due to gravity, } g)] \\ &\quad \times (\text{volume of water displaced, } V) \\ &= [(1000 \text{ kg } m^{-3}) \times (9.81 \text{ m } s^{-2})] \times V \end{aligned}$$

Volume of water displaced, V

$$\begin{aligned} &= \text{volume of portion of rectangular pontoon immersed in water} \\ &= (\text{Area of base of rectangular pontoon}) \times \\ &\quad (\text{depth of portion of rectangular pontoon immersed in water}) \\ &= (lb)Z \\ &= (10 \text{ m} \times 4 \text{ m})Z \\ &= 40Z \end{aligned}$$

$$\begin{aligned} \text{Hence, } F_B &= [(1000 \text{ kg } m^{-3}) \times (9.81 \text{ m } s^{-2})] \times 40Z \\ &= 393600 Z \end{aligned}$$

For stable equilibrium of the system (rectangular pontoon with steel tube placed longitudinally on the deck in a central position), the buoyant force, F_B , must be equal to the combined weight of the rectangular pontoon and the steel tube at its top. Let W be the weight of the rectangular pontoon. Let W_1 be the weight of the steel tube. Hence, we have,

$$\begin{aligned} F_B &= W + W_1 = 280 \text{ kN} + 34 \text{ kN} = 314 \text{ kN} \\ \Rightarrow F_B &= 314 \text{ kN} = 393600 Z \\ \Rightarrow Z &= \frac{314 \times 10^3 \text{ N}}{393600} = 0.798 \text{ m} \end{aligned}$$

Depth of immersion of the buoy, $Z = 0.798 \text{ m}$

The centre of buoyancy B will be located at the centre of gravity of the displaced water, so that, $\overline{OB} = \frac{Z}{2} = \frac{0.798}{2} = 0.399 \text{ m}$

$$\overline{BM} = \frac{I}{V} = \frac{\left(\frac{lb^3}{12}\right)}{(bl)Z} = \frac{b^2}{12Z} = \frac{4^2}{12 \times 0.798} = 1.671 \text{ m}$$

Metacentric height, $\overline{G'M} = \overline{BM} - \overline{BG'}$

$$\overline{BG'} = \overline{OG'} - \overline{OB}$$

$$\overline{OG'} = Z + 0.25 = 0.798 + 0.25 = 1.048 \text{ m}$$

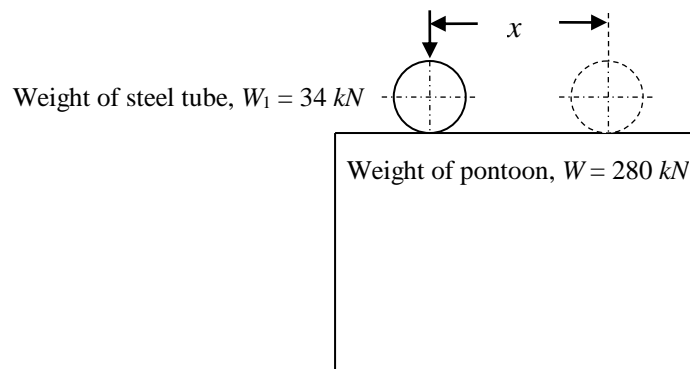
$$\overline{OB} = 0.399 \text{ m}$$

$$\text{Hence, } \overline{BG'} = 1.048 \text{ m} - 0.399 \text{ m} = 0.649 \text{ m}$$

Now, Metacentric height, $\overline{G'M} = \overline{BM} - \overline{BG'} = 1.671 \text{ m} - 0.649 \text{ m} = 1.022 \text{ m}$

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Overturning moment due to movement of steel tube of weight W_1 across the rectangular pontoon is given by (W_1x) . Let x be the maximum distance the tube may be rolled laterally across the deck if the angle of heel is not to exceed 5° . Let the angle of heel be represented by θ .

Metacentric height $\overline{G'M} = 1.022 \text{ m}$

The total weight of the vessel including the weight of steel tube is $(W + W_1) = 314 \text{ kN} = 314 \times 10^3 \text{ N}$

Righting moment = $(W + W_1) \times \overline{G'M} \times \theta$

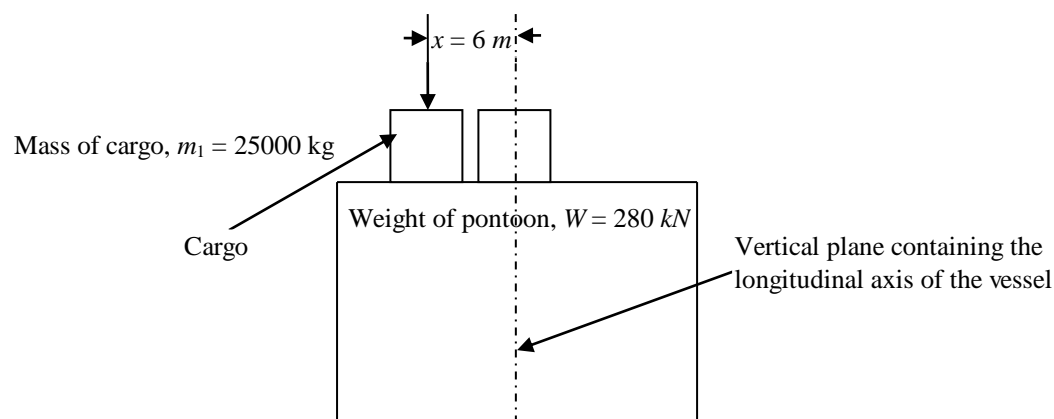
For equilibrium in the tilted position, the righting moment must equal the overturning moment so that, we have,

$$(W + W_1) \times \overline{G'M} \times \theta = W_1x$$

$$\Rightarrow x = \frac{(W + W_1)(\overline{G'M})\theta}{W_1} = \frac{314 \times 10^3 \times 1.022 \times 5^\circ}{34 \times 10^3} = 0.824 \text{ m}$$

Example. The shifting of a portion of cargo of mass 25000 kg through a distance of 6 m at right angles to the vertical plane containing the longitudinal axis of a vessel causes it to heel through an angle of 5° . The displacement of the vessel is 5000 metric tonnes and the value of I is 5840 m^4 . The density of sea water is 1025 kg m^{-3} . Find (a) the metacentric height and (b) the height of the centre of gravity of the vessel above the centre of buoyancy.

Solution.



Displacement of the vessel = 5000 metric tonnes = 5000 x 1000 kg_f = 5000000 kg_f = 5000000 x 9.81 N = 49050000 N = 49050 x 10³ kN

Weight of sea water displaced = 49050 x 10³ kN

Volume of sea water displaced = (Weight of sea water displaced) / (Specific weight of sea water)

$$= \frac{4905000 \text{ N}}{(1025 \times 9.81) \text{ N m}^{-3}} = 4978 \text{ m}^3$$

$$\overline{BM} = \frac{I}{V} = \frac{5840 \text{ m}^4}{4978 \text{ m}^3} = 1.173 \text{ m}$$

Mass of cargo, $m_1 = 25000 \text{ kg}$

$$\begin{aligned} \text{Weight of portion of cargo shifted, } W_1 &= (\text{mass of cargo, } m_1) \times \\ &\quad (\text{acceleration due to gravity, } g) \\ &= 25000 \text{ kg} \times 9.81 \text{ m s}^{-2} \\ &= 245250 \text{ N} \end{aligned}$$

Let the weight of vessel including the weight of portion of cargo shifted be represented as W ; We have determined the value of W to be 49050000 N

Overturning moment due to shifting of a portion of cargo of weight $W_1 = 245250 \text{ N}$ is $(W_1 x) = 245250 \text{ N} \times 6 \text{ m} = 1471500 \text{ N m}$

Let \overline{GM} be the metacentric height of the vessel with the cargo on it and $W = 49050000 \text{ N}$ is the total weight of the vessel including the entire cargo.

$$\text{Righting moment} = W \times \overline{GM} \times \theta$$

where, θ is the angle of tilt caused by the portion of cargo shifted by a distance of $x = 6 \text{ m}$ at right angles to the vertical plane containing the longitudinal axis of a vessel. In this case, θ is given to be $5^\circ = 5^\circ \left(\frac{\pi}{180^\circ} \right)$ radians

For equilibrium in the tilted position, the righting moment must equal the overturning moment so that, we have,

$$W \times \overline{GM} \times \theta = W_1 x$$
$$\Rightarrow \overline{GM} = \frac{W_1 x}{W \theta} = \frac{245250 \text{ N} \times 6 \text{ m}}{49050000 \text{ N} \times \left[5^\circ \left(\frac{\pi}{180^\circ} \right) \right]} = 0.344 \text{ m}$$

The metacentric radius $\overline{BM} = 1.173 \text{ m}$

Height of the centre of gravity, G , of the vessel above the centre of buoyancy, B , is given by $\overline{BG} = \overline{BM} - \overline{GM} = 1.173 \text{ m} - 0.344 \text{ m} = 0.829 \text{ m}$

??????? A rectangular tank 900 mm long and 600 mm wide is mounted on bearings so that it is free to turn on a longitudinal axis. The tank has a mass of 68 kg and its centre of gravity is 150 mm above the bottom. When the tank is slowly filled with water it hangs in stable equilibrium until the depth of water is 450 mm after which it becomes unstable. How far is the axis of the bearings above the bottom of the tank?

Solution.

A cylindrical buoy 1.35 m in diameter and 1.8 m high has a mass of 770 kg. Show that it will not float with its axis vertical in sea water of density 1025 kg m^{-3} .

Solution.

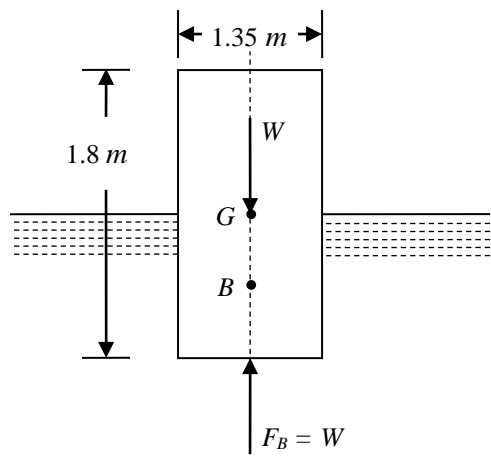


Figure (c)